2010 秋 電磁気

(1) $\boldsymbol{E} = -\mathrm{grad}\, V$ より

$$= -\frac{q}{4\pi\varepsilon_0} \frac{-(\boldsymbol{r}-\boldsymbol{r}_0)}{|\boldsymbol{r}-\boldsymbol{r}_0|^3}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{\boldsymbol{r}-\boldsymbol{r}_0}{|\boldsymbol{r}-\boldsymbol{r}_0|^3}$$

$$\frac{\partial}{\partial x}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}_0|}$$

$$= \frac{\partial}{\partial x}\left\{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right\}^{-\frac{1}{2}}$$

$$= -\frac{1}{2}\frac{2(x-x_0)}{\left\{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right\}^{\frac{3}{2}}}$$

$$= -\frac{\boldsymbol{r}-\boldsymbol{r}_0}{|\boldsymbol{r}-\boldsymbol{r}_0|^3}$$

(2) 位置Qのおける電位は

$$V = \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{(x^2+y^2+(z-a)^2)^{\frac{1}{2}}} - \frac{1}{(x^2+y^2+(z+a)^2)^{\frac{1}{2}}}\right\}$$

$a \ll r$ より

$$V = \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{(x^2+y^2+z^2-2az+a^2)^{\frac{1}{2}}} - \frac{1}{(x^2+y^2+z^2+2az+a^2)^{\frac{1}{2}}}\right\}$$

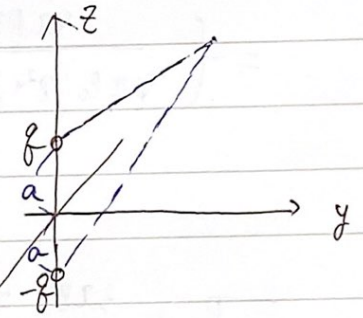
$$= \frac{q}{4\pi\varepsilon_0 r}\left\{\frac{1}{\left(1-\frac{2az}{r^2}+\frac{a^2}{r^2}\right)^{\frac{1}{2}}} - \frac{1}{\left(1+\frac{2az}{r^2}+\frac{a^2}{r^2}\right)^{\frac{1}{2}}}\right\}$$

$$\fallingdotseq \frac{q}{4\pi\varepsilon_0 r}\left\{1-\frac{1}{2}\left(-\frac{2az}{r^2}+\frac{a^2}{r^2}\right) - \left\{1-\frac{1}{2}\left(\frac{2az}{r^2}+\frac{a^2}{r^2}\right)\right\}\right\}$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left\{\frac{2az}{r^2}\right\} \qquad a = \frac{b}{2} \text{ より}$$

$$= \frac{qb}{4\pi\varepsilon_0 r^3}z$$

$\boldsymbol{p} = q\boldsymbol{b}$ より $\quad V = \dfrac{qa-z}{4\pi\varepsilon_0 r^3} = \dfrac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\varepsilon_0 r^3}$



(3) $\vec{E} = -\mathrm{grad}\, V$

$$= -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \frac{q}{4\pi\varepsilon_0 (z-d)}$$

$P = (0, 0, P)$ に注意して.

$$\vec{E} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times \frac{q}{4\pi\varepsilon_0 (z-d)}$$

$$= \left\{ \frac{3xqz}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{\frac{5}{2}}}, \frac{3yqz}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{\frac{5}{2}}}, \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{(x^2+y^2+z^2)^{\frac{3}{2}}} - \frac{3}{2}(x^2+y^2+z^2)^{-\frac{5}{2}} \times 2z \right) \right\}$$

$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{r^2}{r^5} - \frac{3}{2} \frac{r^2}{r^5} \right)$$

$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{1}{r^3} - \frac{3z^2}{r^5} \right)$$

$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{z}{r^3} - \frac{3z^2}{r^5} \right)$$

$$= \frac{1}{4\pi\varepsilon_0} \left[\frac{(1-d)\vec{r}}{r^3} - \vec{P} \right]$$

$$\vec{r} - \vec{P} = (0, 0, P) - (x, y, z)$$

$$= Pz$$