2011 秋 電磁気

(1)

$$4\pi r^2 E = \frac{0}{\varepsilon_0} \qquad E = 0 \qquad (r<a)$$

$$4\pi r^2 E = \frac{q}{\varepsilon_0} \qquad E = \frac{q}{4\pi\varepsilon_0 r^2} \qquad (r>a)$$

(2)

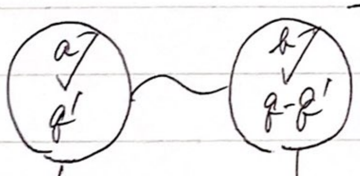
$$U = \int \frac{\varepsilon_0}{2} E^2 dV = \int_a^\infty \frac{\varepsilon_0}{2}\left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{\varepsilon_0}{2}\left(\frac{q}{4\pi\varepsilon_0}\right)^2 \cdot 4\pi \int_a^\infty \frac{1}{r^2} dr$$

$$= \frac{\varepsilon_0}{\cancel{2}}\left(\frac{q}{4\pi\varepsilon_0}\right)^2 \cancel{4}^{2}\pi \left[-\frac{1}{r}\right]_a^\infty$$

$$= \frac{q^2}{8\pi\varepsilon_0 a}$$

(3)



$$U_A = \frac{q'^2}{8\pi\varepsilon_0 a} \qquad U_B = \frac{(q-q')^2}{8\pi\varepsilon_0 b}$$

$$U = \frac{1}{8\pi\varepsilon_0}\left\{\frac{q'^2}{a} + \frac{(q-q')^2}{b}\right\}, \quad \frac{dU}{dq'} = \frac{1}{8\pi\varepsilon_0}\left\{\frac{2q'}{a} - \frac{2(q-q')}{b}\right\} = 0$$

$$\frac{\cancel{2}q'}{a} = \frac{\cancel{2}(q-q')}{b}, \quad \left(\frac{1}{a}+\frac{1}{b}\right)q' = \frac{q}{b}, \quad q' = \frac{q}{b\left(\frac{1}{a}+\frac{1}{b}\right)}$$

$$U = \frac{1}{8\pi\varepsilon_0}\left[\frac{q^2}{ab^2\left(\frac{1}{a}+\frac{1}{b}\right)^2} + \frac{1}{b}\left\{q\left(1-\frac{1}{b\left(\frac{1}{a}+\frac{1}{b}\right)}\right)\right\}^2\right]$$

Date

$$U = \frac{q^2}{8\pi\varepsilon_0 b}\left\{\frac{1}{ab\left(\frac{1}{a}+\frac{1}{b}\right)} + \left(1 - \frac{1}{b\left(\frac{1}{a}+\frac{1}{b}\right)}\right)^2\right\}$$

(4) $$V_a = -\int_\infty^a \frac{q'}{4\pi\varepsilon_0 r^2} = \frac{q'}{4\pi\varepsilon_0 a} = \frac{q}{4\pi\varepsilon_0 ab\left(\frac{1}{a}+\frac{1}{b}\right)}$$

$$V_b = -\int_\infty^b \frac{q-q'}{4\pi\varepsilon_0 r^2} = \frac{q-q'}{4\pi\varepsilon_0 b} = \frac{q}{4\pi\varepsilon_0 b}\left\{1 - \frac{1}{b\left(\frac{1}{a}+\frac{1}{b}\right)}\right\}$$

$$\frac{1}{ab\left(\frac{1}{a}+\frac{1}{b}\right)} = \frac{1}{b}\left\{1 - \frac{1}{b\left(\frac{1}{a}+\frac{1}{b}\right)}\right\}$$ を示せればよい

$$(\mathrm{RHS}) = \frac{1}{b}\left\{\frac{b\left(\frac{1}{a}+\frac{1}{b}\right)}{b\left(\frac{1}{a}+\frac{1}{b}\right)} - \frac{1}{b\left(\frac{1}{a}+\frac{1}{b}\right)}\right\}$$

$$= \frac{1}{b}\left\{\frac{\frac{b}{a}}{b\left(\frac{1}{a}+\frac{1}{b}\right)}\right\}$$

$$= \frac{1}{b}\,\frac{1}{a\left(\frac{1}{a}+\frac{1}{b}\right)} = (\mathrm{LHS})$$

よって $V_a = V_b$