Date

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(1) $\varepsilon_n = \left(n + \frac{1}{2}\right) h\nu$

$$Z(\beta) = \sum_{n=0}^{\infty} e^{-\beta\left(n+\frac{1}{2}\right)h\nu}$$

$$= e^{-\beta h\nu} \sum \left(e^{-\beta h\nu}\right)^n$$

$$= e^{-\frac{\beta h\nu}{2}} \frac{1}{1 - e^{-\beta h\nu}}$$

$$= \frac{1}{e^{\frac{\beta h\nu}{2}} - e^{-\frac{\beta h\nu}{2}}} = \frac{1}{2}\left(\sinh\frac{\beta h\nu}{2}\right)^{-1}$$

(2) $U = -\frac{\partial}{\partial\beta} \log\left(2\sinh\frac{\beta h\nu}{2}\right)^{-1}$

$$= \frac{\partial}{\partial\beta} \log 2\sinh\frac{\beta h\nu}{2}$$

$$= \frac{1}{2\sinh\frac{\beta h\nu}{2}} \times 2\cosh\frac{\beta h\nu}{2} \times \frac{h\nu}{2} = \frac{h\nu}{2}\left(\tanh\frac{\beta h\nu}{2}\right)^{-1}$$

(3) $U_0 = \lim_{\beta\to\infty} U(\beta) = \frac{h\nu}{2} \lim_{\beta\to\infty} \frac{1}{\tanh\frac{\beta h\nu}{2}}$

$$= \frac{h\nu}{2} \times 1 = \frac{h\nu}{2}$$

(4) $\Delta U(\beta) = \frac{h\nu}{2}\left\{\left(\tanh\frac{\beta h\nu}{2}\right)^{-1} - 1\right\}$

$$= \frac{h\nu}{2}\left(\frac{e^{\frac{\beta h\nu}{2}} + e^{-\frac{\beta h\nu}{2}}}{e^{\frac{\beta h\nu}{2}} - e^{-\frac{\beta h\nu}{2}}} - \frac{e^{\frac{\beta h\nu}{2}} - e^{-\frac{\beta h\nu}{2}}}{e^{\frac{\beta h\nu}{2}} - e^{-\frac{\beta h\nu}{2}}}\right)$$

$$= \frac{h\nu}{\cancel{2}} \frac{\cancel{2} e^{-\frac{\beta h\nu}{2}}}{e^{\frac{\beta h\nu}{2}} - e^{-\frac{\beta h\nu}{2}}}$$

$$= h\nu \frac{1}{e^{\beta h\nu} - 1} = h\nu \cdot f^{(0)}(h\nu)$$

1粒子あたりのエネルギーが
hνであるようなボソンで
化学potentialが0.
流入流出がない
エネルギー密度