2012 春 統計

(1) $W = {}_N C_n = \dfrac{N!}{(N-n)!\,n!}$

(2) $S = k_B \log \dfrac{N!}{(N-n)!\,n!}$

(3) $S = k_B\{N\log N - \cancel{N} - (N-n)\log(N-n) + \cancel{N-n} - n\log n + \cancel{n}\}$

$= k_B\{N\log N - n\log n - (N-n)\log(N-n)\}$

(4) $\dfrac{1}{T} = \left(\dfrac{\partial S}{\partial E}\right)$

$\dfrac{\partial S}{\partial E} = \dfrac{\partial S}{\partial n}\dfrac{\partial n}{\partial E}$

$\dfrac{\partial S}{\partial n} = k_B\{-\log n - \cancel{1} + \log(N-n) - (N-n)\cancel{\dfrac{1}{(N-n)}} \times -1\}$

$= k_B \log\dfrac{N-n}{n}$

$\dfrac{\partial n}{\partial E} = \dfrac{1}{\varepsilon}$

$\dfrac{\partial S}{\partial E} = \dfrac{1}{\varepsilon}\cdot k_B\log\dfrac{N-n}{n} = \dfrac{1}{T}$

$\log\dfrac{N-n}{n} = \dfrac{\varepsilon}{k_B T}$

$\dfrac{N-n}{n} = e^{\frac{\varepsilon}{k_B T}}$

$N-n = n e^{\frac{\varepsilon}{k_B T}}$ $\quad n(1+e^{\frac{\varepsilon}{k_B T}}) = N$

$n = \dfrac{N}{1+e^{\frac{\varepsilon}{k_B T}}}$

$\dfrac{n}{N} = \dfrac{1}{1+e^{\frac{\varepsilon}{k_B T}}}$

(5) $E = n\varepsilon$

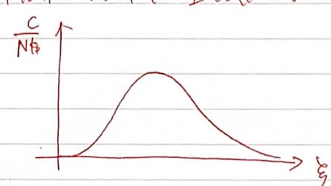
$= \dfrac{\varepsilon N}{1+e^{\frac{\varepsilon}{k_B T}}}$

$C = \dfrac{\partial E}{\partial T} = \varepsilon N\left(1+e^{\frac{\varepsilon}{k_B T}}\right)^{-1\,\prime} = -\varepsilon N\left(1+e^{\frac{\varepsilon}{k_B T}}\right)^{-2} \times -\dfrac{\varepsilon}{k_B T^2} e^{\frac{\varepsilon}{k_B T}}$

$= \dfrac{\varepsilon^2 N}{k_B T^2}\dfrac{e^{\frac{\varepsilon}{k_B T}}}{\left(1+e^{\frac{\varepsilon}{k_B T}}\right)^2}$

$\lim_{T\to 0} C = 0$

2準位系はショットキー型比熱となる.



$\dfrac{C}{Nk_B} = \dfrac{1}{\xi^2}\dfrac{e^{\frac{1}{\xi}}}{\left(1+e^{\frac{1}{\xi}}\right)^2}$ $\quad \left(\because \xi = \dfrac{k_B T}{\varepsilon}\right)$