Date

2018 秋期 統計力学

$_NC_n = \frac{_NP_n}{n!}$

(1) $_NC_n = \frac{N!}{n!(N-n)!}$

スターリング
$N! = \frac{n^n}{e^n}$

(2) $S = k_B \ln W = k_B(\ln N! - \ln n! - \ln(N-n)!)$

$= k_B(N\ln N - N - n\ln n + n - (N-n)\ln(N-n) + (N-n))$

$= k_B(N\ln N - n\ln n - N\ln(N-n) + n\ln(N-n))$

$= k_B\left(N\ln\frac{N}{N-n} + n\ln\frac{N-n}{n}\right)$

(3) 1コの原子を結晶内部から表面に移動させるのに必要なエネルギー ε.

$E = n\varepsilon$

$\frac{\partial S}{\partial E} = \frac{\partial S}{\partial n}\frac{\partial n}{\partial E} = \frac{1}{\varepsilon}\frac{\partial S}{\partial n}$

$= \frac{1}{\varepsilon}k_B\left(-\log n - n\cdot\frac{1}{n} - N\cdot\frac{1}{N-n}\cdot -1 + \ln(N-n) + n\cdot\frac{1}{N-n}\cdot -1\right)$

$= \frac{k_B}{\varepsilon}\left(-\ln n - 1 + \frac{N}{N-n} + \ln(N-n) - \frac{n}{N-n}\right)$

足して0

$\frac{1}{T} = \frac{k_B}{\varepsilon}(\log(N-n) - \ln n)$

$\ln\frac{N-n}{n} = \frac{\varepsilon}{k_BT}$

$\frac{N}{n} = e^{\frac{\varepsilon}{k_BT}} + 1 \rightarrow \therefore \bar{n}(T) = \frac{N}{e^{\frac{\varepsilon}{k_BT}}+1}$

(4) $p = \frac{\bar{n}}{N}$

$$= \frac{1}{e^{\frac{\varepsilon}{kT}} + 1}$$ … Fermi distribution