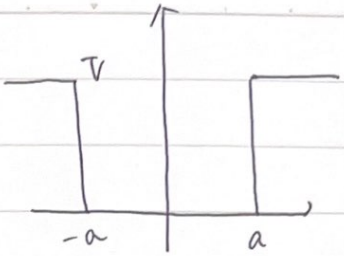
2018 着量子

(1)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$$

$$\psi'' = -\frac{2mE}{\hbar^2}\psi \qquad \left(k^2 = \frac{2mE}{\hbar^2}\right)$$



$$\psi = A\sin kx + B\cos kx$$

$$\psi(a) = A\sin ka + B\cos ka = 0$$

$$\psi(-a) = -A\sin ka + B\cos ka = 0$$

$$2B\cos ka = 0$$

$B = 0$ のとき $A \neq 0$.

$$\psi(x) = A\sin kx$$

$$\psi(a) = A\sin ka = 0$$

$$ka = n\pi, \quad k = \frac{n\pi}{a}$$

$$\int_{-a}^{a}\psi^*\psi\,dx = |A|^2\int_{-a}^{a}\sin^2 kx = |A|^2 a = 1$$

$$|A|^2 = \frac{1}{a}, \quad A = \sqrt{\frac{1}{a}}$$

$$E = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2$$

$$\psi(x) = \sqrt{\frac{1}{a}}\sin kx$$

$$ka = \frac{2n+1}{2}\pi \quad \text{のとき} \quad (n = 0, 1, 2\cdots)$$

$$k = \frac{2n+1}{2a}\pi$$

$$\psi(a) = A\sin\frac{(2n+1)}{2}\pi = 0$$

$$A(-1)^n = 0 \qquad A = 0.$$

$$\psi(x) = \sqrt{\frac{1}{a}}\cos\frac{2n+1}{2a}\pi x.$$

(2) $p^2 = \frac{2mE}{\hbar^2}$, $p^2 = \left(\frac{2n+1}{2a}\pi\right)^2$

$$E = \frac{\hbar^2}{2m}\left(\frac{2n+1}{2a}\pi\right)^2$$

(3) $\psi_0(x) = \sqrt{\frac{1}{a}}\cos\frac{\pi x}{2a}$

(4) $\langle p^2\rangle = \int \psi_0^* \left(-i\hbar\frac{\partial}{\partial x}\right)^2 \psi_0\, dx$

$$= -\hbar^2 \int \sqrt{\frac{1}{a}}\cos\frac{\pi x}{2a}\,\frac{\partial^2}{\partial x^2}\sqrt{\frac{1}{a}}\cos\frac{\pi x}{2a}\,dx$$

$$= -\frac{\hbar^2}{a}\int \cos\frac{\pi x}{2a} \times \left(\frac{\pi}{2a}\right)^2 - \cos\frac{\pi x}{2a}\,dx$$

$$= \frac{\hbar^2}{a}\left(\frac{\pi}{2a}\right)^2 \int_{-a}^{a} \cos^2\frac{\pi x}{2a}\,dx$$

$$= \frac{\hbar^2\pi^2}{4a^3}\int_{-a}^{a} \frac{1+\cos\frac{\pi x}{a}}{2}\,dx$$

$$= \frac{\hbar^2\pi^2}{4a^2} \qquad \Delta p = \sqrt{\langle p^2\rangle} = \frac{\hbar\pi}{2a}$$

(5) $\langle x^2\rangle = \int \frac{1}{a}\cos\frac{\pi x}{2a}\, x^2 \cos\frac{\pi x}{2a}\,dx$

$$= \frac{1}{a}\int x^2 \cos\frac{\pi x}{2a}\,dx = \frac{1}{a}\left[x^2\frac{2a}{\pi}\right.$$

$$= \frac{1}{a}\int x^2\,\frac{1+\cos\frac{\pi x}{a}}{2}\,dx = \frac{1}{a}\int \frac{x^2}{2} + \frac{1}{2}x^2\cos\frac{\pi x}{a}\,dx$$

$$= \frac{1}{a}\left[\frac{1}{6}x^3\right]_{-a}^{a} + \frac{1}{2a}\int_{-a}^{a} x^2\cos\frac{\pi x}{a}\,dx$$

$$= \frac{a^2}{3} + \frac{1}{2a}\left[x^2\frac{a}{\pi}\sin\frac{\pi x}{a}\right] - \int 2x\frac{a}{\pi}\sin\frac{\pi x}{a}\,dx$$

$$= \frac{a^2}{3} - \int_{-a}^{a} \frac{x}{\pi} \sin\frac{\pi x}{a}\, dx$$

$$= \frac{a^2}{3} + \left[\frac{x}{\pi}\frac{a}{\pi}\cos\frac{\pi x}{a}\right]_{-a}^{a} - \int \frac{1}{\pi}\frac{a}{\pi}\cos\frac{\pi x}{a}\, dx$$

$$= \frac{a^2}{3} + \frac{a^2}{\pi^2}(-1) - \frac{-a^2}{\pi^2}(-1) - \int_{0}^{a} \frac{2a}{\pi^2}\cos\frac{\pi x}{a}\, dx$$

$$= \frac{a^2}{3} - \frac{2a^2}{\pi^2} - \left[\frac{2a}{\pi^2}\frac{a}{\pi}\sin\frac{\pi x}{a}\right]_{0}^{a}$$

$$= \frac{a^2}{3} - \frac{2a^2}{\pi^2}$$

$$\Delta x = \sqrt{\frac{a^2}{3} - \frac{2a^2}{\pi^2}}$$

$$\Delta x \cdot \Delta p = \sqrt{\frac{a^2}{3} - \frac{2a^2}{\pi^2}} \times \frac{\hbar\pi}{2a}$$

$$= \sqrt{\frac{\hbar^2\pi^2}{4a^2}\left(\frac{a^2}{3} - \frac{2a^2}{\pi^2}\right)}$$

$$= \sqrt{\frac{\hbar^2\pi^2}{12} - \frac{\hbar^2}{2}}$$

xとpは同時観測不可

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