

20<sup>18</sup> 暑期 物理数学

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{-\frac{x^2}{2}} H_n(x)$$

$A_n$

$$(1) x H_n(x) = n H_{n-1}(x) + \frac{1}{2} H_{n+1}(x)$$

$$\psi_n(x) = A_n H_n(x)$$

$$H_n(x) = \frac{\psi_n(x)}{A_n}$$

$x \psi_n(x)$  与

$$\psi_{n-1}(x) \pm \psi_{n+1}(x) \text{ 与 } \frac{1}{2} \psi_{n+1}(x)$$

$$x \frac{\psi_n(x)}{A_n} = n \frac{\psi_{n-1}(x)}{A_{n-1}} + \frac{1}{2} \frac{\psi_{n+1}(x)}{A_{n+1}}$$

$H_n \quad H_{n-1} \quad H_{n+1}$

$$x \psi_n(x) = \sqrt{\frac{n}{2}} \psi_{n-1}(x) + \sqrt{\frac{n+1}{2}} \psi_{n+1}(x)$$

$$(2) \frac{d}{dx} \psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} \left( -x e^{-\frac{x^2}{2}} H_n(x) + e^{-\frac{x^2}{2}} \frac{d}{dx} H_n(x) \right)$$

$$\frac{d}{dx} H_n(x) = x H_n(x) + n H_{n-1}(x) - \frac{1}{2} H_{n+1}(x)$$

$$= \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} \left( -x e^{-\frac{x^2}{2}} H_n(x) + e^{-\frac{x^2}{2}} \left( x H_n(x) + n H_{n-1}(x) - \frac{1}{2} H_{n+1}(x) \right) \right)$$

$$H_n(x) = \frac{\psi_n(x)}{A_n} \text{ 方便整理}$$

$$\frac{d}{dx} \psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{-\frac{x^2}{2}} \left( n \frac{\psi_{n-1}(x)}{A_{n-1}} - \frac{1}{2} \frac{\psi_{n+1}(x)}{A_{n+1}} \right)$$

$A_n$

$$= n \frac{A_n}{A_{n-1}} \psi_{n-1}(x) - \frac{1}{2} \frac{A_n}{A_{n+1}} \psi_{n+1}(x)$$

$$= \sqrt{\frac{n}{2}} \psi_{n-1}(x) - \sqrt{\frac{n+1}{2}} \psi_{n+1}(x)$$

$$(3) \left( \frac{d}{dx} + x \right) \varphi_n(x) = \sqrt{2n} \varphi_{n-1}(x)$$

$$\left( \frac{d}{dx} - x \right) \varphi_n(x) = -\sqrt{2(n+1)} \varphi_{n+1}(x)$$

(4)

$$\left( \frac{d}{dx} - x \right) \underbrace{\left( \frac{d}{dx} + x \right) \varphi_n(x)}_{(3) \text{ の答}} = \left( \frac{d}{dx} - x \right) \left( \sqrt{2n} \varphi_{n-1}(x) \right)$$

$$= \sqrt{2n} \left( \frac{d}{dx} - x \right) \varphi_{n-1}(x)$$

$$= \sqrt{2n} \left( -\sqrt{2n} \right) \varphi_n(x) = \boxed{-2n \varphi_n(x)}$$

$$\left( \frac{d^2}{dx^2} - x^2 \right) \varphi_n(x) = -2n \varphi_n(x)$$

$$\left( -\frac{1}{2} \frac{d^2}{dx^2} + \frac{1}{2} x^2 \right) \varphi_n(x) = \boxed{n} \varphi_n(x)$$

固有値

$$\hat{A} \varphi_n(x) = n \varphi_n(x)$$