

2019 中下期 量子力学

(1) エルミート演算子 \hat{A}, \hat{B} が同時固有状態が存在しないことを示す。

$$\hat{A}|\psi\rangle = a_0|\psi\rangle \quad \text{と仮定}$$

$$\begin{aligned} \circ \langle\psi|\hat{A}\hat{B}|\psi\rangle &= \sum_k \langle\psi|\hat{A}|k\rangle \langle k|\hat{B}|\psi\rangle \\ &= \sum_k a_0^* \delta_{0k} \langle k|\hat{B}|\psi\rangle = a_0^* \langle\psi|\hat{B}|\psi\rangle \end{aligned}$$

$$\begin{aligned} \circ \langle\psi|\hat{B}\hat{A}|\psi\rangle &= \sum_k \langle\psi|\hat{B}|k\rangle \langle k|\hat{A}|\psi\rangle \\ &= \sum_k a_k \delta_{kj} \langle\psi|\hat{B}|k\rangle = a_j \langle\psi|\hat{B}|\psi\rangle \end{aligned}$$

$$\circ (a_0^* - a_j) \langle\psi|\hat{B}|\psi\rangle \neq 0 \quad \dots \quad \text{かつ } \delta_{ij} \quad \text{かな?}$$

$$\begin{cases} i=j & a_0 \neq 0 & 0 = \text{かつ} \\ i \neq j & a_0 \neq a_j & (a_0 - a_j) \langle\psi|\hat{B}|\psi\rangle = 0 \end{cases} \quad \text{同じ同時固有状態が存在しない}$$

$$\begin{aligned} (2) \langle\chi|\chi\rangle &= \langle\phi|(\hat{A} - i\kappa\hat{B})(\hat{A} + i\kappa\hat{B})|\phi\rangle & |\chi\rangle &= (\hat{A} + i\kappa\hat{B})|\phi\rangle \\ &= \langle\phi|\hat{A}^2 + i\kappa\hat{A}\hat{B} - i\kappa\hat{B}\hat{A} + \kappa^2\hat{B}^2|\phi\rangle \\ &= \langle\phi|\hat{A}^2 - \kappa\hbar + \kappa^2\hat{B}^2|\phi\rangle \\ &= \kappa^2 \langle\phi|\hat{B}^2|\phi\rangle - \kappa\hbar + \langle\phi|\hat{A}^2|\phi\rangle \end{aligned}$$

$$(3) \langle\chi|\chi\rangle \geq 0 \quad \therefore f(\kappa) = \langle\phi|\hat{B}^2|\phi\rangle \kappa^2 - \kappa\hbar + \langle\phi|\hat{A}^2|\phi\rangle$$

$$\circ D = \hbar^2 - 4 \langle\phi|\hat{B}^2|\phi\rangle \langle\phi|\hat{A}^2|\phi\rangle \leq 0$$

$$\langle\phi|\hat{B}^2|\phi\rangle \langle\phi|\hat{A}^2|\phi\rangle \geq \frac{\hbar^2}{4}$$

$$(4) \sqrt{\langle\phi|\hat{B}^2|\phi\rangle \langle\phi|\hat{A}^2|\phi\rangle} \geq \frac{\hbar}{2}$$

$$(5) \langle\phi|\hat{B}^2|\phi\rangle = 0 \quad \text{ならば} \quad \langle\phi|\hat{A}^2|\phi\rangle = \infty$$

$$\langle\phi|\hat{A}^2|\phi\rangle = 0 \quad \text{ならば} \quad \langle\phi|\hat{B}^2|\phi\rangle = \infty$$

これは、不確定性原理を示している。