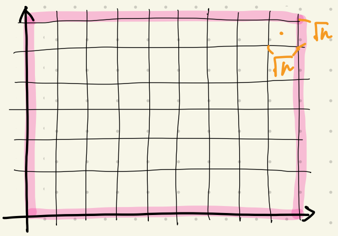
2020 中大 秋期

IV

$$\mathcal{H} = \frac{p^2}{2m} + \alpha x$$

(1) 絶対温度Tの時の粒子1コの分配関数

$$Z_1 = \frac{1}{h}\int_{-\infty}^{\infty} dp \int_0^{x} dx' \, e^{\frac{-\mathcal{H}}{k_BT}}$$



$$Z_1 = \frac{1}{h}\int_{-\infty}^{\infty} e^{-\frac{p^2}{2mk_BT}} dp \int_0^{x} e^{-\frac{\alpha x'}{k_BT}} dx'$$

$$= \frac{1}{h}\sqrt{2mk_BT\pi}\left[-\frac{k_BT}{\alpha} e^{-\frac{\alpha x'}{k_BT}}\right]_0^x$$

$$= \frac{1}{h}\sqrt{2mk_BT\pi} \times -\frac{k_BT}{\alpha}\left(e^{-\frac{\alpha x}{k_BT}} - 1\right)$$

$$= \left(\frac{2mk_BT\pi}{h^2}\right)^{\frac{1}{2}}\left\{\frac{k_BT\left(1 - e^{-\frac{\alpha x}{k_BT}}\right)}{\alpha}\right\}$$

(2) 粒子Nコの分配関数 Z_N

$$\frac{1}{N!} Z_1^{N} = Z_N$$

(3) ヘルムホルツの自由エネルギーを求めよ

$$F = -k_BT \log Z_N$$

$$= -k_BT\left\{\frac{1}{2}\log\left(\frac{2mk_BT\pi}{h^2}\right) + \log\left(\frac{k_BT\left(1 - e^{-\frac{\alpha x}{k_BT}}\right)}{\alpha}\right) - \log N!\right\}$$

(4) 粒子の数密度のx依存性 $n(x)$ を計算せよ.

$$n(x) = \frac{N}{x}$$

(5) xでの圧力 P

$$P = -\frac{\partial F}{\partial V} = -\frac{\partial F}{\partial x}$$

$$= -(-k_BT)\frac{\partial}{\partial x}\log\left(\frac{k_BT(1-e^{-\frac{\alpha x}{k_BT}})}{\alpha}\right)$$

$$= \cancel{k_BT}\cdot\frac{\alpha}{\cancel{k_BT}(1-e^{-\frac{\alpha x}{k_BT}})}\cdot\frac{\cancel{k_BT}}{\cancel{\alpha}}\cdot\frac{\cancel{\alpha}}{\cancel{k_BT}}e^{-\frac{\alpha x}{k_BT}}$$

$$= \frac{\alpha\, e^{-\frac{\alpha x}{k_BT}}}{(1-e^{-\frac{\alpha x}{k_BT}})}$$