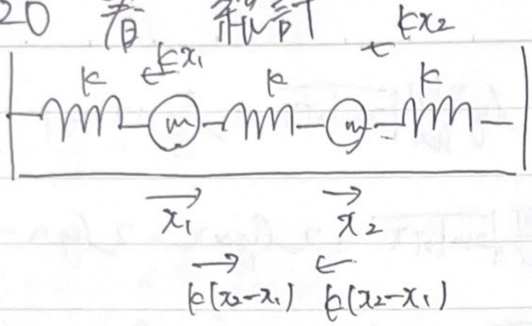
2020 看 統計

(1)



$$m\ddot{x}_1 = k(x_2-x_1) - kx_1$$
$$m\ddot{x}_2 = -k(x_2-x_1) - kx_2$$

$$\begin{cases} X = \dfrac{mx_1+mx_2}{m+m} = \dfrac{x_1+x_2}{2} \\ x = x_2 - x_1 \end{cases}$$ とする.

$$m(\ddot{x}_1+\ddot{x}_2) = -k(x_1+x_2)$$
$$m\ddot{X} = -kX$$
$$m(\ddot{x}_2-\ddot{x}_1) = -2k(x_2-x_1) - k(x_2-x_1)$$
$$m\ddot{x} = -3kx.$$

$$\begin{cases} \ddot{X} = -\dfrac{k}{m}X \\ \ddot{x} = -\dfrac{3k}{m}x \end{cases} \longrightarrow H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2 x_1^2 + \frac{1}{2}m\omega^2 x_2^2$$

$$Z_1 = \frac{1}{h}\int e^{-\frac{H}{k_BT}}\,dp\,dx$$
$$= \frac{1}{h}\int e^{-\frac{p_1^2}{2mk_BT}} e^{-\frac{p_2^2}{2mk_BT}} e^{-\frac{\frac{1}{2}m\omega^2 x_1^2}{k_BT}} e^{-\frac{\frac{1}{2}m\omega^2 x_2^2}{k_BT}}\,dp_1\,dp_2\,dx_1\,dx_2$$
$$= \frac{1}{h}\left(\sqrt{2mk_BT\pi}\right)^2 + \left(\sqrt{\frac{2k_BT}{m\omega^2}}\right)^2 = \frac{4}{h}\left(2mk_BT\pi\right)\left(\frac{2k_BT}{m\omega^2}\right)$$

$$= \frac{16 k_B^2 T^2 \pi}{h\omega^2}$$

$$Z_1^N = Z_N = \left(\frac{16 k_B^2 T^2 \pi}{h\omega^2}\right)^N$$

比熱は $\frac{\partial U}{\partial T}$

$$U = k_B T^2 \frac{\partial}{\partial T} \log Z_N = k_B T^2 \frac{\partial}{\partial T}\left(N \log T^2\right)$$

$$= N k_B T^2 \frac{1}{T^2} \times 2T = 2N k_B T.$$

$$\frac{\partial U}{\partial T} = 2N k_B$$

(2)
(3) kに依存していないので 比熱は変わらず $2Nk_B$