2021 春期 中大 量子力学

$$\hat{l}_x = \frac{1}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$\hat{l}_y = \frac{1}{i}\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$

$$[\hat{l}_x, \hat{l}_y] = -\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) + (z_x - x_z)(y_z - z_y)$$

$$= -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y} = \underline{i\hat{l}_z}$$

$$[\hat{l}^2, \hat{l}_z] = [\hat{l}_x^2 + \hat{l}_y^2 + \hat{l}_z^2, \hat{l}_z]$$

$$= l_x\underbrace{[l_x, l_z]}_{-il_y} + \underbrace{[l_x, l_z]}_{-il_y}l_x + l_y\underbrace{[l_y, l_z]}_{il_x} + \underbrace{[l_y, l_z]}_{il_x}l_y$$

$$= -il_xl_y - il_yl_x + il_yl_x + il_xl_y$$

$$= 0.$$

l^2 と l_z は同時観測可能だが、l_z が確定すると l_x と l_y は不確定。

$$[\hat{H}, \hat{x}] = \frac{p^2}{2m}x - x\frac{p^2}{2m} = \frac{1}{2m}(p^2x - xp^2)$$

$$= \frac{1}{2m}\{p\underbrace{[p, x]}_{-i\hbar} + \underbrace{[p, x]}_{-i\hbar}p\}$$

$$= -\frac{i}{m}\hbar p$$

$$p = -i\hbar\frac{\partial}{\partial x}$$

$$[H, \hat{p}]\psi = [V, p]\psi = \{Vp\psi - p(\psi V)\}$$

$$= Vp\cancel{\psi} - p\cancel{\psi}V - pV\psi = \underline{i\hbar\frac{\partial V}{\partial x}}$$

$$\hat{l}_z\psi_\mu = \mu\psi_\mu \qquad \hat{l}_\pm = \hat{l}_x \pm i\hat{l}_y$$

$$\hat{l}_z\hat{l}_\pm\psi = \{[l_z, l_\pm] + l_\pm l_z\}\psi$$

$$= (\pm\hat{l}_\pm)\psi + l_\pm\mu\psi$$

$$= \underline{(\mu\pm1)\hat{l}_\pm\psi}$$

$$[l_z, l_\pm]$$

$$= [l_z, l_x \pm il_y]$$

$$= \underbrace{[l_z, l_x]}_{il_y} \pm i\underbrace{[l_z, l_y]}_{-il_x}$$

$$= \pm(l_x \pm il_y)$$

$$= \pm l_\pm$$

$$l_z\psi_{\mu\pm1} = \mu\pm1\,\psi_{\mu\pm1}$$

$$[H, l_z] = \left[\frac{p^2}{2m} + V, l_z\right]$$

$$= \underbrace{[V, l_z]}_{=0} + \frac{1}{2m}[p_j^2, l_z]$$

$$= \frac{1}{2m}\{p_j[p_j, l_z] + [p_j, l_z]p_j\}$$

$$= \frac{1}{2m}\{\hbar \sum_{jl} \varepsilon_{zjl}(p_j p_l + p_l p_j)\}$$

$$= \frac{\hbar}{2m}(p \times p)_z = 0$$

$$[p_j, l_z]$$

$$= \sum_{kl} i\,\varepsilon_{zkl}[p_j, r_k p_l]$$

$$= \sum_{kl} i\,\varepsilon_{zkl}[p_j, r_k]p_l$$

$$= \sum_{kl} i\,\varepsilon_{zkl}(-i\hbar\,\delta_{jk})p_l$$

$$= \sum_{l} i\,\varepsilon_{zjl}(-i\hbar)p_l$$

$$= \hbar \sum_{l} \varepsilon_{zjl} p_l .$$

$$(a \times b)_i = \sum_j \sum_k \varepsilon_{ijk} a_j b_k$$

$$V_{eff} = \frac{r \cdot p}{2\pi r^3} = \frac{p}{2\pi r^2}$$

$$\frac{1}{i}\left[V, x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right]\psi$$

$$= \frac{1}{i}\left[\cancel{Vx\frac{\partial\psi}{\partial y}} - \underline{x\frac{\partial V}{\partial y}\psi} - \cancel{xV\frac{\partial\psi}{\partial y}} - \cancel{Vy\frac{\partial\psi}{\partial x}} + \underline{y\frac{\partial V}{\partial x}\psi} + \cancel{yV\frac{\partial\psi}{\partial x}}\right]$$

$$= \frac{1}{i}\left\{y\frac{\partial V}{\partial x}\psi - x\frac{\partial V}{\partial y}\psi\right\}$$

$$= \frac{1}{i}\left(-\frac{xy}{r^2}V\psi + \frac{xy}{r^2}V\psi\right)$$

$$= 0$$

$$\boxed{\begin{aligned} \frac{\partial V}{\partial x} &= \frac{x}{r}\frac{\partial}{\partial r}\left(\frac{p}{2\pi r^2}\right) \\ &= \frac{xp}{2\pi r}\left(-\frac{1}{r^3}\right) \\ &= -\frac{x}{r^2}V \end{aligned}}$$