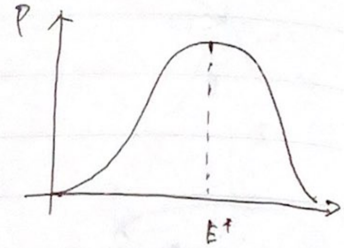
【2021 春期 統計力学】



(1) $P = C E^{\frac{3N}{2}-1} e^{-\frac{E}{k_B T}}$

$P' = C E^{\frac{3N}{2}-2} e^{-\frac{E}{k_B T}} \left\{ \frac{3N}{2} - 1 - \frac{E}{k_B T} \right\} = 0$ のとき.

$E^* = \left(\frac{2N}{2} - 1 \right) k_B T$

$$\boxed{\begin{array}{l} \frac{E}{k_B T} = x \\ \frac{1}{k_B T} dE = dx \end{array}}$$

(2) $\frac{1}{C} = \int_0^\infty E^{\frac{3N}{2}-1} e^{-\frac{E}{k_B T}} dE$

$= \int (k_B T x)^{\frac{3N}{2}-1} e^{-x} k_B T \, dx$

$= (k_B T)^{\frac{3N}{2}} \int x^{\frac{3N}{2}-1} e^{-x} dx$

$= \underline{(k_B T)^{\frac{3N}{2}} I\left(\frac{3N}{2} - 1\right).}$ //

(3) $I_n = \left[x^n \cdot (-e^{-x}) \right]_0^\infty - \int n (x)^{n-1} (-e^{-x}) dx$

$= n \int x^{n-1} e^{-x} dx$

$= \underline{n I(n-1)}$ //

(4) $I(0) = \int_0^\infty e^{-x} dx = -\left[e^{-x} \right]_0^\infty = 1$

$I(1) = 1 \cdot I(0) = 1$

$I(2) = 2 I(1) = 2 \quad \rightarrow n!$

$C = (k_B T)^{-\frac{3N}{2}} \frac{1}{\left(\frac{3N}{2} - 1\right)!}$

(5) $\langle E \rangle = \int_0^\infty C E^{\frac{3N}{2}} e^{-\frac{E}{k_B T}} dE = C (k_B T)^{\frac{3N}{2}+1} \int x^{\frac{3N}{2}} e^{-x} dx$

$= k_B T \cdot \frac{1}{\left(\frac{3N}{2} - 1\right)!} \left(\frac{3N}{2}\right)!$

$= \underline{\frac{3}{2} N k_B T}$ //

(6)

$$\lim_{N\to\infty} \frac{\langle E\rangle}{E^*} = \frac{\frac{3}{2}Nk_BT}{\left(\frac{3N}{2}-1\right)k_BT} = 1.$$

E^* … 最も確からしい値

$S = k\ln W$ が Max になる

久保 . P239 問18.