2021 秋 電磁気

1.

$a < r < b$ のとき

ガウスの法則より

$$E(r) \cdot 2\pi r = \frac{\lambda}{\varepsilon_0}$$

$$\therefore E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\therefore \mathbf{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r} \mathbf{e}_r$$

2.

$$V_0 = -\int_b^a \mathbf{E}\, d\mathbf{r}$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \int_a^b \frac{1}{r}\, dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \log \frac{b}{a}$$

$$\therefore \lambda = \frac{2\pi\varepsilon_0}{\log \frac{b}{a}} V_0$$

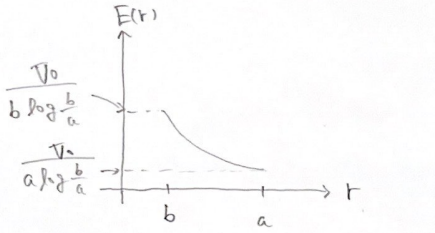
3.

(1) $a < r < b$ のとき

$$\mathbf{E}(r) = \frac{1}{2\pi\varepsilon_0 r} \cdot \frac{2\pi\varepsilon_0}{\log \frac{b}{a}} V_0 \mathbf{e}_r$$

$$= \frac{V_0}{r \log \frac{b}{a}} \mathbf{e}_r$$

$r < b$, $a < r$ では電場は 0



(2) $a < r < b$ のとき

$$V(r) = -\int_b^r \mathbf{E}\, d\mathbf{r}$$

$$= \frac{V_0}{\log \frac{b}{a}} \int_r^b \frac{1}{r'}\, dr$$

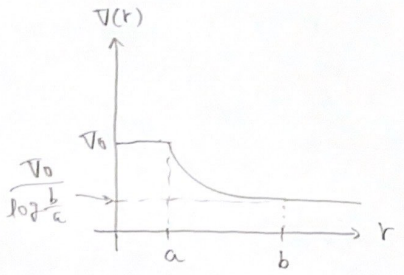
$$= \frac{V_0}{\log \frac{b}{a}} \log \frac{b}{r}$$

$r > b$ のとき

$$V(b) = \frac{V_0}{\log \frac{b}{a}}$$

$r < a$ のとき

$$V(a) = V_0$$



4. $Q = CV$ より

$$\frac{2\pi\varepsilon_0}{\log \frac{b}{a}} V_0 = C \cdot V_0$$

$$\therefore C = \frac{2\pi\varepsilon_0}{\log \frac{b}{a}}$$

5. $a < r < b$ のとき

$$U_e(r) = \frac{\varepsilon_0}{2} E^2 = \frac{\varepsilon_0}{2} \left(\frac{V_0}{r \log \frac{b}{a}} \right)^2$$

$r < a$, $b < r$ では

$$U_e = 0$$

6.

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \cdot \frac{2\pi\varepsilon_0}{\log \frac{b}{a}} \cdot V_0^2$$

$$= \frac{\pi\varepsilon_0}{\log \frac{b}{a}} V_0^2$$

(別解)

$$U = \frac{Q^2}{2C} = \frac{1}{2} \cdot \frac{\log \frac{b}{a}}{2\pi\varepsilon_0} \cdot \left(\frac{2\pi\varepsilon_0}{\log \frac{b}{a}} V_0 \right)^2$$

$$= \frac{\pi\varepsilon_0}{\log \frac{b}{a}} V_0^2$$ 同じになる

7.

$0 < r < a$ のとき

$$B(r) \cdot 2\pi r = \mu_0 \cdot j\pi r^2$$

$$= \mu_0 \frac{I}{\pi a^2} \cdot \pi r^2$$

$$= \mu_0 \frac{I}{a^2} r^2$$

$$\therefore B(r) = \frac{\mu_0 I r}{2\pi a^2} e_\theta$$

$a < r < b$ のとき

$$B(r) \cdot 2\pi r = \mu_0 I$$

$$\therefore B(r) = \frac{\mu_0 I}{2\pi r} e_\theta$$

$b < r$ のとき

$$B(r) = 0$$

8. 単位長さあたりの磁束は

$$d\Phi = B \cdot dr$$

$$\Phi = \int d\Phi = \int_0^b B\, dr$$

$$= \int_0^a \frac{\mu_0 I}{2\pi a^2} r\, dr + \int_a^b \frac{\mu_0 I}{2\pi} \frac{1}{r} dr$$

$$= \frac{\mu_0 I}{2\pi a^2} \cdot \frac{a^2}{2} + \frac{\mu_0 I}{2\pi} \log \frac{b}{a}$$

$$= \frac{\mu_0}{2\pi} \left(\log \frac{b}{a} + \frac{1}{2} \right) I$$

$\Phi = LI$ と比較して

$$L = \frac{\mu_0}{2\pi} \left(\log \frac{b}{a} + \frac{1}{2} \right)$$