2022 秋 量子力学

1. $\psi(x,t) = \phi(x) e^{-i\frac{E}{\hbar}t}$ とすると

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2}\cancel{e^{-i\frac{E}{\hbar}t}} = i\hbar\phi(x)\cdot\left(-i\frac{E}{\hbar}\right)\cancel{e^{-i\frac{E}{\hbar}t}}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} = E\phi(x)$$

2. $$\frac{d^2\phi}{dx^2} = -\frac{2mE}{\hbar^2}\phi(x) = -k^2\phi(x)$$

$\therefore\ \phi(x) = A\sin kx + B\cos kx$

$\phi(0) = 0$ より

$B = 0$

$\phi(L) = 0$ より

$kL = n\pi \quad (n = 1, 2, 3, \cdots)$

$$\Rightarrow \frac{2mE}{\hbar^2} = \left(\frac{n\pi}{L}\right)^2$$

$$\therefore\ E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 \quad (n = 1, 2, 3, \cdots)$$

$$\therefore\ \phi_n(x) = A\sin\frac{n\pi}{L}x$$

規格化すると

$$|A|^2\int_0^L \sin^2\frac{n\pi}{L}x\,dx = |A|^2\int_0^L \frac{1-\cos\frac{2n\pi}{L}x}{2}dx$$

$$= \frac{L}{2}|A|^2 = 1$$

$$\therefore\ A = \sqrt{\frac{2}{L}}$$

$$\therefore\ \phi_n = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x \quad (n = 1, 2, 3, \cdots)$$

3. $\phi_0 = \sqrt{\frac{2}{L}}\sin\frac{\pi}{L}x$, $\phi_1 = \sqrt{\frac{2}{L}}\sin\frac{2\pi}{L}x$

$$\psi_{01}(x,t) = \sqrt{\frac{1}{L}}\left(e^{-i\frac{E_0}{\hbar}t}\sin\frac{\pi}{L}x + e^{-i\frac{E_1}{\hbar}t}\sin\frac{2\pi}{L}x\right)$$

$$\langle P \rangle = \int_0^L \psi_{01}^* \, P \, \psi_{01} \, dx$$

$$= \int_0^L \psi_{01}^* \left(-i\hbar \frac{d}{dx}\right) \psi_{01} \, dx$$

$$= -\frac{i\hbar}{L} \int_0^L \left(e^{i\frac{E_0}{\hbar}t} \sin\frac{\pi x}{L} + e^{i\frac{E_1}{\hbar}t} \sin\frac{2\pi x}{L} \right)$$

$$\left(\frac{\pi}{L} e^{-i\frac{E_0}{\hbar}t} \sin\frac{\pi x}{L} + \frac{2\pi}{L} e^{-i\frac{E_1}{\hbar}t} \sin\frac{2\pi}{L}x \right) dx$$

波動関数の直交性

↓

$$= -\frac{i\hbar}{L} \int_0^L \left\{ \frac{\pi}{L} \sin^2\frac{\pi}{L}x + \frac{2\pi}{L} \sin^2\frac{2\pi}{L}x \right\} dx$$

$$= -\frac{i\hbar}{L} \left(\frac{\pi}{L} \cdot \frac{L}{2} + \frac{2\pi}{L} \cdot \frac{L}{2} \right)$$

$$= -\frac{i\hbar}{L} \left(\frac{\pi}{2} + \pi \right)$$

$$= -i\frac{3\pi\hbar}{2}$$