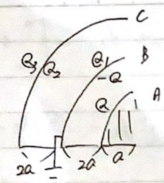
2016 東工大 材料 電磁気

(1) B表, BC裏, C表の電荷を

Q_1, Q_2, Q_3 とする.



$4\pi r^2 E = \frac{Q}{\varepsilon_0}$, $E = \frac{Q}{4\pi r^2 \varepsilon_0}$ $(a<r<3a)$

$4\pi r^2 E = \frac{Q_1}{\varepsilon_0}$, $E = \frac{Q_1}{4\pi r^2 \varepsilon_0}$ $(3a<r<5a)$

$4\pi r^2 E = \frac{Q_2+Q_3+Q_1}{\varepsilon_0}$, $E = \frac{Q_1+Q_2+Q_3}{4\pi r^2 \varepsilon_0}$ $(r \geqq 5a)$

$$V_B = -\int_{\infty}^{5a} \frac{Q_1+Q_2+Q_3}{4\pi r^2 \varepsilon_0} dr - \int_{5a}^{3a} \frac{Q_1}{4\pi r^2 \varepsilon_0} dr = 0.$$

C上でのガウスの法則を考えると.

$$4\pi r^2 \cdot 0 = \frac{Q_2+Q_1}{\varepsilon_0} \quad \therefore Q_2 = -Q_1$$

またCの表, 裏で電荷の和は0になるので.

$$\therefore Q_3 = -Q_2 . \Rightarrow Q_3 = +Q_1$$

$$V_B = -\int_{\infty}^{5a} + \frac{Q_1}{4\pi r^2 \varepsilon_0} dr - \int_{5a}^{3a} \frac{Q_1}{4\pi \varepsilon_0 r^2} dr = 0$$

$$-\frac{Q_1}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{5a} - \frac{Q_1}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{5a}^{3a} = 0$$

$$-\frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{3a}\right) = 0$$

$$Q_1 = 0 \quad \therefore Q_2, Q_3 = 0.$$

$$E = \begin{cases} \frac{Q}{4\pi r^2 \varepsilon_0} & (0<r<3a) \\ 0 & (\text{otherwise}) \end{cases}$$

(2) $U_{AB} = \frac{1}{2} Q V_{AB}$, $U_{BC} = \frac{1}{2} 0 V_{BC} = 0$

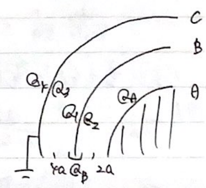
$$V_{AB} = -\int_{3a}^{a} \frac{Q}{4\pi r^2 \varepsilon_0} dr = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{3a}^{a}$$

$$= \frac{Q}{6\pi\varepsilon_0 a}$$

$$U_{AB} = \frac{1}{2} Q \frac{Q}{6\pi\varepsilon_0 a} = \frac{Q^2}{12 a \pi \varepsilon_0}$$

(3) $E(2a) = E(4a)$

電場を考える.



$4\pi r^2 E = \frac{Q_A}{\varepsilon_0}$, $E = \frac{Q_A}{4\pi\varepsilon_0 r^2}$ $(a<r<3a$ ~~$r<a$~~$)$

$4\pi r^2 E = \frac{Q_1-Q_2+Q_A}{\varepsilon_0}$ $E = \frac{Q_1+Q_2+Q_A}{4\pi\varepsilon_0 r^2}$ $(3a<r<5a)$

$3Q_A$ Q_A

B上で $Q_A = -Q_2$ $\therefore Q_2 = -Q_A$

~~$2Q_A$~~ Q_A

$4Q_A$ $-Q_A$

~~$4\pi r^2 E = \frac{}{\varepsilon_0}$~~ $E(2a) = \frac{Q_A}{4\pi\varepsilon_0(2a)^2}$

$E(4a) = \frac{Q_1 \cancel{+2Q_A}}{4\pi\varepsilon_0(4a)^2}$

$Q_B = Q_1 + Q_2$

$Q_1 = Q_B - Q_2$

$= Q_B + Q_A$

$E(2a) = E(4a)$ より

$\frac{Q_1+2Q_A}{4} = Q_A$

$\frac{3}{4}Q_A - \frac{1}{4}Q_A = -\frac{1}{4}Q_B$

~~$Q_A = Q_B$~~

$\frac{1}{4}(Q_A+Q_B) = Q_A$

$Q_A + Q_B = 4Q_A$

$3Q_A = Q_B$ $\frac{Q_B}{Q_A} = 3$

(4) $V_C = -\int_{\infty}^{5a} \frac{Q_4+Q_3+Q_1}{4\pi\varepsilon_0 r^2}\,dr = 0$

C上の電場は

$4\pi r^2 \cdot 0 = \frac{Q_3+Q_1}{\varepsilon_0}$ $\therefore Q_3 = -Q_1$ $Q_1 = 4Q_A$ より

$Q_3 = -4Q_A$

$V_C = -\frac{Q_4}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{5a} = 0$

$\frac{Q_4}{4\pi\varepsilon_0}\frac{1}{5a} = 0$ $\therefore Q_4 = 0$

C上に蓄えられた電荷は $Q_3 + Q_4 = \underline{-4Q_A}$

(5) $V_{AB} = -\int_{3a}^{a} \frac{Q_A}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q_A}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{3a}\right) = \underline{\frac{Q_A}{6\pi\varepsilon_0 a}}$

$V_{BC} = -\int_{5a}^{3a} \frac{-4Q_A}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q_A}{\pi\varepsilon_0}\left(-\frac{1}{3a}+\frac{1}{5a}\right)$

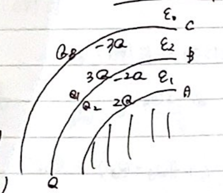
$= \frac{Q_A}{\pi\varepsilon_0}\left(-\frac{5}{15a}+\frac{3}{15a}\right)$

$= -\frac{2Q_A}{15a\pi\varepsilon_0}$ 大きさとして $\underline{\frac{2Q_A}{15a\pi\varepsilon_0}}$

(6) $2Q + Q_2 = 0$

$Q_2 = -2Q.$

$Q_1 = Q - (-2Q) = 3Q$



$4\pi r^2 E = \frac{2Q}{\varepsilon_0}$; $E = \frac{Q}{2\pi\varepsilon_1}\frac{1}{r^2}$ $(a<r<3a)$

$4\pi r^2 E = \frac{3Q}{\varepsilon_0}$, $E = \frac{3Q}{4\pi\varepsilon_{02} r^2}$ $(3a<r<5a)$

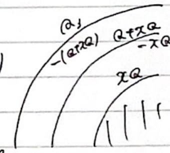
$E(2a) = \frac{Q}{2\pi\varepsilon_{01}(2a)^2} = \frac{3Q}{4\pi\varepsilon_{02}(4a)^2} = E(4a)$

$\frac{1}{\varepsilon_1} = \frac{3}{2\times 4\varepsilon_2}$, $\underline{\frac{\varepsilon_2}{\varepsilon_1} = \frac{3}{8}}$

(7) $V_{AB} = -\int_{3a}^{a} \frac{Q}{2\pi\varepsilon_1 r^2}\,dr = \frac{Q}{2\pi\varepsilon_1}\left[\frac{1}{a}-\frac{1}{3a}\right) = \underline{\frac{Q}{3\pi\varepsilon_1 a}}$

$V_{BC} = -\int_{5a}^{3a} \frac{3Q}{4\pi\varepsilon_2 r^2}\,dr = \frac{3Q}{4\pi\varepsilon_2}\left[\frac{1}{3a}-\frac{1}{5a}\right] = \underline{\frac{Q}{10\pi\varepsilon_2 a}}$

(8) $4\pi r^2 E = \frac{xQ}{\varepsilon_{01}}$, $E = \frac{xQ}{4\pi\varepsilon_{01} r^2}$ $(a<r<3a)$



$4\pi r^2 E = \frac{(1+x)Q}{\varepsilon_{02}}$, $E = \frac{(1+x)Q}{4\pi\varepsilon_2 r^2}$ $(3a<r<5a)$

$V_{AB} = -\int_{3a}^{a} \frac{xQ}{4\pi\varepsilon_{01} r^2}\,dr = \frac{xQ}{4\pi\varepsilon_1}\left[\frac{1}{a}-\frac{1}{3a}\right] = \frac{xQ}{6\pi\varepsilon_1 a}$

$V_{BC} = -\int_{5a}^{3a} \frac{(1+x)Q}{4\pi\varepsilon_2 r^2}\,dr = \frac{(1+x)Q}{4\pi\varepsilon_2}\left[\frac{1}{3a}-\frac{1}{5a}\right] = \frac{(1+x)Q}{30\pi\varepsilon_2 a}$

$V_{AB} = V_{BC}$ より $\frac{xQ}{6\pi\varepsilon_1 a} = \frac{(1+x)Q}{\underset{5}{\cancel{30}}\pi\varepsilon_2 a}$, $\frac{x}{\varepsilon_1} = \frac{(1+x)}{5\varepsilon_2}$

$\frac{\varepsilon_2}{\varepsilon_1} = \frac{(1+x)}{5x} = \frac{3}{8}$, $8+8x = 15x$

$7x = 8$, $\underline{x = \frac{8}{7}}$

$U_{AB} = \frac{1}{2}\frac{8}{7}Q\frac{Q}{6\pi\varepsilon_1 a}\frac{8}{7}$

$U_{BC} = \frac{1}{2}\left(1+\frac{8}{7}\right)Q\frac{\left(1+\frac{8}{7}\right)Q}{\underset{5}{\cancel{30}}\pi\varepsilon_2 a}$

$\frac{U_{BC}}{U_{AB}} = \left(1+\frac{8}{7}\right)^2\frac{1}{5\varepsilon_2}\times\left(\frac{7}{8}\right)^2\varepsilon_1 = \left(\frac{15}{7}\right)^2\frac{8}{3}\cdot\frac{1}{5}\left(\frac{7}{8}\right)^2 = \underline{\frac{15}{8}}$