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1 [B]

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty t^{-\frac{1}{2}} e^{-t}\, dt$$

$t = x^2 \qquad t\ 0 \to \infty$

$\qquad\qquad x\ 0 \to \infty$

$dt = 2x\,dx$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty x^{-1} e^{-x^2} \cdot 2x\,dx$$

$$= \int_0^\infty 2e^{-x^2}\,dx \qquad \int_0^\infty e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$$

$$= 2\sqrt{\pi} \times \frac{1}{2}$$

$$= \sqrt{\pi}$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$$

自力

(3)

$$\Gamma(n+1) = \int_0^\infty e^{-t} t^n\, dt = \int_0^\infty (-e^{-t})' t^n\, dt = \underbrace{\left[-e^{-t} t^n\right]_0^\infty}_{\to 0} - \int_0^\infty -e^{-t} n t^{n-1}\, dt$$

$$= n\int_0^\infty e^{-t} t^{n-1}\, dt$$

$$= n\Gamma(n)$$

$$= n(n-1)\Gamma(n-1)$$

$$= n(n-1)(n-2)\cdots\Gamma(1)$$

$$= n!$$

$$\Gamma(n) = (n-1)!$$

$$\Gamma\left(n+\frac{1}{2}\right) = \left(n-\frac{1}{2}\right)\left(n-\frac{3}{2}\right)\cdots\frac{3}{2}\cdot\frac{1}{2}\,\boxed{\Gamma\left(\frac{1}{2}\right)}$$

$$= (2n-1)(2n-3)\cdots 3\cdot 1\cdot\sqrt{\pi}\cdot 2^{-n}\,\boxed{\sqrt{\pi}}$$

項の数は

$5.4.3.2.1 \to 5-1+1 = 5$ の様に

$n-\frac{1}{2}-\frac{1}{2}+1 = n$ コ

$$= (2n-1)!!\,2^{-n}\sqrt{\pi}$$

$$\frac{2n(2n-1)(2n-2)\cdots 2\cdot 1}{2n\cdot(2n-2)\cdots 2} = \frac{(2n)!}{n!}2^{-n}$$

$$Ans = \frac{(2n)!}{n!}2^{-n}2^{-n}\sqrt{\pi} \neq$$

$$= \underline{\frac{(2n)!}{n!}2^{-2n}\sqrt{\pi}}$$

(4) $\lim_{z\to -n}(z+n)\Gamma(z)$ $\qquad x>0,\ \mathrm{Re}\,z<0$

$\Gamma(n+1) = \Gamma(n)\times n$

$\Gamma(z) = \frac{1}{z}\Gamma(z+1)$

$$\int_0^\infty e^{-t}t^{ai+b-1}dt$$

$$= \underbrace{\left[-e^{-t}t^{b-1+ai}\right]_0^\infty}_{0} + \int_0^\infty e^{-t}(ai+b-1)t^{ai+b-2}dt$$

$= ,\ldots$

複素積分がマジに出来ない.

$\Gamma(z) = \frac{1}{z}\Gamma(z+1)$ $\qquad \mathrm{Re}\,z>-1,\ \mathrm{Re}\,z\neq 0$

$= \frac{1}{z}\frac{1}{(z+1)}\Gamma(z+2)$ $\qquad \mathrm{Re}\,z>-2\ \ \mathrm{Re}\,z\neq 0$

$= \frac{1}{z(z+1)\cdots(z+n)}\Gamma(z+n+1)$ $\qquad z>-(n+1)$

$z\neq 0,-1,\cdots,-n$

$$\lim_{z\to -n}(z+n)\Gamma(z) = \lim_{z\to -n}\cancel{(z+n)}\frac{\Gamma(z+n+1)}{z(z+1)\cdots\cancel{(z+n)}}$$

$$= \lim_{z\to -n}\frac{\Gamma(z+n+1)}{z(z+1)\cdots(z+n-1)}$$

$$= \frac{1}{-n(-n+1)\cdots(-1)}\Gamma(1) = \frac{(-1)^n}{n(n-1)\cdots 1}\Gamma(1) = \frac{(-1)^n}{n!}$$

[c] $\dfrac{\partial^2}{\partial t^2} f(r,t) = v^2 \dfrac{1}{r^2} \dfrac{\partial}{\partial r} \left\{ r^2 \dfrac{\partial}{\partial r} f(r,t) \right\}$

(5) 上の一般解を求めよ

$$f = \frac{1}{r} u(r,t)$$

$$r^2 \frac{\partial}{\partial r} f = r^2 \frac{\partial}{\partial r}\left(\frac{u}{r}\right) = r^2\left(-\frac{u}{r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

$$= -u + r\frac{\partial u}{\partial r}$$

$$\frac{\partial}{\partial r}\left(-u + r\frac{\partial u}{\partial r}\right) = -\frac{\partial u}{\partial r} + \frac{\partial u}{\partial r} + r\frac{\partial^2 u}{\partial r^2}$$

$$= r\frac{\partial^2 u}{\partial r^2}$$

$$\therefore \frac{1}{r}\frac{\partial^2}{\partial t^2} u = v^2 \frac{1}{r}\frac{\partial^2 u}{\partial r^2}$$

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial r^2}$$

$$u = a(r - vt) + b(r + vt)$$

$$f = \frac{u}{r} = \frac{1}{r}\{a(r - vt) + b(r + vt)\}$$

$$\frac{d}{dt}\left(\frac{1}{r}\{a(r-vt) + b(r+vt)\}\right)$$

(6) $f(r, t=0) = \dfrac{1}{r}\{a(r) + b(r)\} = \dfrac{\sin kr}{r}$

$$\left.\frac{\partial f}{\partial t}\right|_{t=0} = \frac{v}{r}\{-a(r) + b(r)\} = 0 \leftrightarrow a = b$$

$$\rightarrow \frac{2a}{r} = \frac{\sin k}{r}$$

$$a = \frac{\sin kr}{2}$$

$$f(r,t) = \frac{1}{2r}\{\sin k(r - vt) + \sin k(r + vt)\}$$