2021年　都立大学　冬期　力学

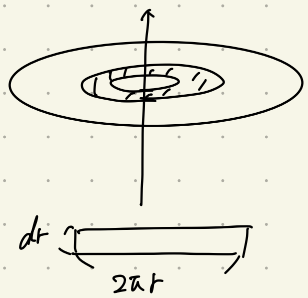
(1)

$$I_G = \int 2\pi r\, dr\, \sigma\, r^2$$

$$= 2\pi\sigma \int_0^R r^3\, dr$$

$$= 2\pi \frac{M}{R^2\pi}\left[\frac{1}{4}R^4\right]$$

$$= \frac{1}{2}MR^2$$



$$\sigma = \frac{M}{R^2\pi}$$

(2)

$$\begin{cases} I\theta'' = fR \\ Mx'' = Mg\sin\alpha - f \\ R\theta'' = x'' \end{cases}$$

$$f = \frac{I}{R}\theta''$$

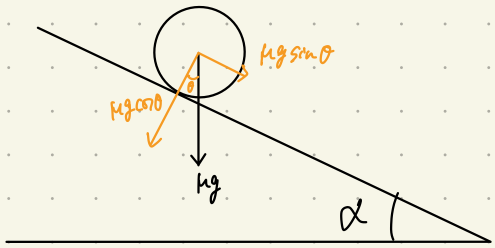
$$Mx'' = Mg\sin\alpha - \frac{I}{R}\theta''$$

$$Mx'' + \frac{1}{2}MR^2\frac{x''}{R^2} = Mg\sin\alpha$$

$$\frac{3}{2}Mx'' = Mg\sin\alpha$$

$$x'' = \frac{2g}{3}\sin\alpha$$

$$\frac{dv}{dt} = \frac{2g}{3}\sin\alpha$$



$$dv = \frac{2g}{3}\sin\alpha\, dt$$

$$v = \frac{2g}{3}\sin\alpha\cdot t + C$$

$$v(0) = C = 0$$

$$v = \frac{2g}{3}\sin\alpha\cdot t$$

(3)

$$\begin{cases} I\theta'' = \mu' Mg\cos\alpha\cdot R \\ Mx'' = Mg\sin\alpha - \mu' Mg\cos\alpha \\ R\theta'' \geq x'' \end{cases}$$ (滑らない条件式)

$$\frac{1}{2}\cancel{M}R^{\cancel{2}}\theta'' = \mu'\cancel{M}g\cos\alpha\cdot\cancel{R}$$

$$\begin{cases} \theta'' = \frac{2\mu' g}{R}\cos\alpha \\ x'' = g\sin\alpha - \mu' g\cos\alpha \end{cases}$$

代入

$$2\mu' g\cos\alpha \geq g\sin\alpha - \mu' g\cos\alpha$$

$$2\mu' \geq \tan\alpha - \mu' \quad \therefore \tan\alpha \leq 3\mu'$$

(4) $\tan\alpha = 3\mu'$ のときの t'

$$dv = (g\sin\alpha - \mu' g\cos\alpha)\, dt$$

$$v = (g\sin\alpha - \mu' g\cos\alpha)t + C$$

$$v(0) = C = V_0$$

$$v = (g\sin\alpha - \mu' g\cos\alpha)t + V_0$$

$$= \cos\alpha\cdot gt(\tan\alpha - \mu') + V_0$$

$$= \cos(\tan^{-1}3\mu')\cdot gt'\cdot 2\mu' + V_0$$

$$\theta' = \left(\frac{2\mu' g}{R}\cos\alpha\right)t + D$$

$$\theta'(t') = \left(\frac{2\mu' g}{R}\cos\alpha\right)t' + D = 0 \qquad D = -\left(\frac{2\mu' g}{R}\cos(\tan^{-1}3\mu')\right)t'$$

$$\theta' = \frac{2\mu' g}{R}\cos(\tan^{-1}3\mu')\, t'$$

$$\therefore \cancel{2\mu' g\cos(\tan^{-1}3\mu')t' - \left(2\mu' g\cos(\tan^{-1}3\mu')\right)t'} = \cos(\tan^{-1}3\mu')\, gt'\cdot\cancel{2\mu'} + V_0$$

$$t' = -\frac{V_0}{g\cos(\tan^{-1}3\mu')}$$