

# 2022年 都立大学 冬期 力学

(1)  $N = \mu g r \sin \theta$

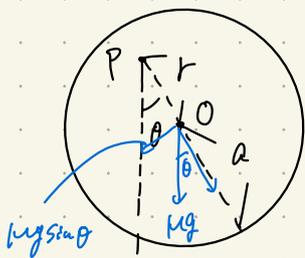
(2)  $I \ddot{\theta} = -\mu g r \sin \theta$

(3)  $\sin \theta \approx \theta$

$$\ddot{\theta} = -\frac{\mu g r}{I} \theta$$

$$\omega = \sqrt{\frac{\mu g r}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{\mu g r}}$$

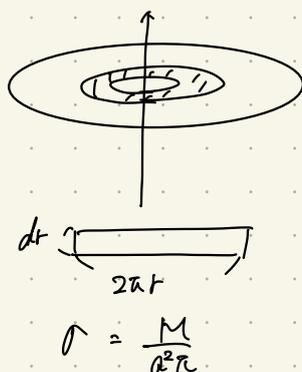


(4)  $I_G = \int 2\pi r dr \sigma r^2$

$$= 2\pi \sigma \int_0^a r^3 dr$$

$$= 2\pi \frac{M}{a^2 \pi} \left[ \frac{1}{4} a^4 \right]$$

$$= \frac{1}{2} M a^2$$



(5) 平行軸の定理より

$$I = I_G + M(r)^2 = \frac{1}{2} M a^2 + M r^2$$

(6)  $T = 2\pi \sqrt{\frac{I}{\mu g r}}$

$$= 2\pi \sqrt{\frac{1}{\mu g r} \left( \frac{1}{2} M a^2 + M r^2 \right)}$$

$$= 2\pi \sqrt{\frac{a^2}{2g r} + \frac{r}{g}}$$

$$f(r) = \frac{a^2}{2g r} + \frac{r}{g} \quad \text{と置}$$

$$\frac{df}{dr} = -\frac{a^2}{2g r^2} + \frac{1}{g} = 0 \quad \frac{a^2}{2g r^2} = \frac{1}{g}$$

r	...	$\frac{a}{\sqrt{2}}$	...
f'	-	0	+
f	↘		↗

$$r^2 = \frac{a^2}{2}$$

$$r = \frac{a}{\sqrt{2}}$$

$$r = \frac{a}{\sqrt{2}} \quad \text{と、最小値をとる}$$

$$\therefore \text{周期は } T = \frac{a}{\sqrt{2}} \quad \text{と、最小値をとる}$$