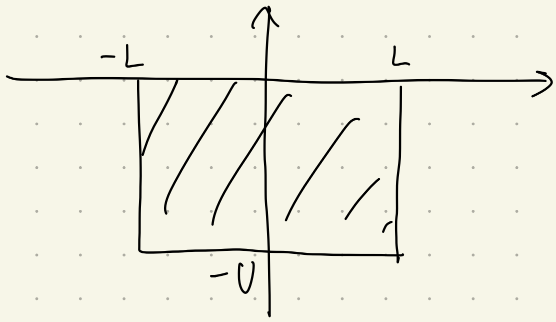
2022年　都立大学　冬期　量子力学

(1)

$$-\frac{\hbar^2}{2m}\varphi'' = (E+U)\varphi$$

$$\varphi'' = -\frac{2m(E+U)}{\hbar^2}\varphi = -k^2\varphi$$

$$\varphi = A\cos kx + B\sin kx$$



(2)

$$-\frac{\hbar^2}{2m}\varphi'' = E\varphi$$

$$\varphi'' = -\frac{2mE}{\hbar^2}\varphi = \alpha\varphi$$

$$\varphi = Ce^{\alpha x} + De^{-\alpha x}$$

(3) $|x| \geq L$ のとき

$$\varphi_I = \begin{cases} Ce^{\alpha x} & (x<-L) \\ De^{-\alpha x} & (x>L) \end{cases}$$

φ が偶関数 より

$$\varphi_{II} = A\cos kx \quad (-L<x<L)$$

接続条件 より

$$\begin{cases} \varphi_I(L) = \varphi_{II}(L) \\ \varphi_I(-L) = \varphi_{II}(-L) \\ \frac{d}{dx}\varphi_I(L) = \frac{d}{dx}\varphi_{II}(L) \\ \frac{d}{dx}\varphi_I(-L) = \frac{d}{dx}\varphi_{II}(-L) \end{cases}$$

$$De^{-\alpha L} = A\cos kL$$

$$Ce^{-\alpha L} = A\cos kL$$

$$-\alpha De^{-\alpha L} = -kA\sin kL$$

$$\alpha Ce^{-\alpha L} = kA\sin kL$$

$$\frac{\alpha Ce^{-\alpha L}}{Ce^{-\alpha L}} = \frac{kA\sin kL}{A\cos kL} \qquad \therefore \alpha = k\tan kL$$

(4) $V(x) = -W\delta(x)$

$$-\frac{\hbar^2}{2m}\varphi_{III}'' = E\varphi_{III} \qquad \varphi_{III} = Ee^{\alpha x} + Fe^{-\alpha x}$$

$$\varphi_{III} = \begin{cases} \sqrt{2\alpha}\,e^{-\alpha x} & (x>0) \\ \sqrt{2\alpha}\,e^{\alpha x} & (x<0) \end{cases}$$

$$|E|^2\int_0^\infty e^{-2\alpha x}dx = |E|^2\left[-\frac{1}{2\alpha}e^{-2\alpha x}\right]_0^\infty$$

$$\Rightarrow |E|^2 = 2\alpha \qquad E = \sqrt{2\alpha}$$

$$|F|^2\int_{-\infty}^0 e^{2\alpha x}dx = |F|^2\left[\frac{1}{2\alpha}e^{2\alpha x}\right]_{-\infty}^0 \qquad \therefore F = \sqrt{2\alpha}$$

(5)

$$-\frac{\hbar^2}{2m}\varphi_{III}'' - W\delta(x)\varphi(x) = E\varphi(x)$$

$$-\frac{\hbar^2}{2m}\left(\varphi_{III}'(\varepsilon) - \varphi_{III}'(-\varepsilon)\right) - W\varphi_{III}(0) = 0$$

$$-\alpha\sqrt{2\alpha}\,e^{-\alpha\varepsilon} - \alpha\sqrt{2\alpha}\,e^{\alpha(\varepsilon)} = -\frac{2mW}{\hbar^2}\sqrt{2\alpha}$$

$$2\alpha = \frac{2mW}{\hbar^2} \qquad \therefore \alpha = \frac{2mW}{\hbar^2}$$

$$\frac{\sqrt{-2mE}}{\hbar} = \frac{2mW}{\hbar^2}$$

$$\to -2mE = \frac{4m^2W^2}{\hbar}$$

$$E = -\frac{2m^2W^2}{m\hbar}$$

$$= -\frac{2mW^2}{\hbar}$$